Frequency Conversion

- Frequency conversion-Hamiltonian \([1]\)
  \[
  \hat{H} = i\hbar \kappa A_p \hat{a}_{in} \hat{a}_{out}^\dagger + \text{h.c.}
  \]

- strong pump field \(A_p\) treated classically
  \[
  \hat{a}_{in}(t) = \hat{a}_{in}(0) \cos(\kappa A_p t) - \hat{a}_{out}(0) \sin(\kappa A_p t)
  \]
  \[
  \hat{a}_{out}(t) = \hat{a}_{out}(0) \cos(\kappa A_p t) + \hat{a}_{in}(0) \sin(\kappa A_p t)
  \]

- complete conversion for
  \[
  \kappa A_p t = \frac{\pi}{2}
  \]

- \(\kappa\) depends on effective nonlinearity \(d_{\text{eff}}\), geometry and mode overlap

Quantum pulse gate

Dispersion-engineered frequency up-conversion

Energy conservation
\[ \omega_p + \omega_{\text{in}} = \omega_{\text{out}} \]

Phasematching
\[ \beta_p + \beta_{\text{in}} + \frac{2\pi}{\Lambda} = \beta_{\text{out}} \]

\[ \beta'_p = \beta'_{\text{in}} \]

long interaction length

Process engineering – Pump pulse

$G(\omega_{\text{in}}, \omega_{\text{out}})$
Process engineering – Pump pulse

\[ G(\omega_{in}, \omega_{out}) \]
Process engineering – Pump pulse
Quantum pulse gate

Dispersion engineered frequency conversion

Bogoliubov transformation: beam splitter

\[
\hat{A}^{(k)}_{\text{red}} \rightarrow \cos(\theta) \hat{A}^{(k)}_{\text{red}} + \sin(\theta) \hat{A}^{(k)}_{\text{green}}
\]

\[
\hat{A}^{(k)}_{\text{green}} \rightarrow \cos(\theta) \hat{A}^{(k)}_{\text{green}} - \sin(\theta) \hat{A}^{(k)}_{\text{red}}
\]
Quantum pulse gate

Dispersion engineered frequency conversion

\[ v_{g}^{(\text{pump})} = v_{g}^{(\text{signal})} \]

efficiency adjusted with pump power
Quantum pulse gate

Dispersion engineered frequency conversion

In-house manufactured periodically poled LN waveguide:

- Crystal length: 15…40 mm
- SFG: 1536nm / 874nm to 557nm
- Poling period 4.4 µm
- Temperature stabilized at 190°C
- Bandwidth compression
Group velocity matching

Ti:Sapphire Laser
80MHz @ 874nm

1550nm CW fiber laser

Andor Shamrock SR-500
Newton EMCCD

Curvature: Group-velocity mismatch

Simultaneous phasematching for spatial modes

SMF to further analysis components

Joint Spectral Intensity

0.08nm
Spectrum before vs. after conversion

Spectrum of converted photons measured on single photon sensitive spectrometer

Internal conversion efficiency: 75%
External conversion efficiency: 17%

Spectrum is changed, but no implication on quantumness / efficiency

Ideal spectral filter: efficiency 13.4%

Allgaier, et al., Nat. Com. 8, 14288 (2016)
Measurement Tomography of QPG

quantifies quality of QPG for TM POVM measurements

\[ n^{\alpha\beta} = \text{tr}(\hat{M}^\alpha \hat{\rho}^\beta) \]

Temporal-mode detector tomography

Temporal-mode detector tomography

Temporal-mode detector tomography

### Ideal measurements

![Graphs of ideal measurements]

### Characterized measurements

![Graphs of characterized measurements]
Temporal-mode tomography

Test on shaped classical pulses

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Idealized projectors</th>
<th>Average fidelity of reconstructed temporal-mode density matrices</th>
</tr>
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<tbody>
<tr>
<td>5</td>
<td>87.9% ± 4.1%</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>81.3% ± 3.1%</td>
<td></td>
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Theory

Experiment:

Requirements for high dimensional quantum coding

- Basis for encoding
- Generation of tailored states
- Manipulation of basis modes
① Engineered parametric downconversion
② Quantum pulse gate
③ Applications
Temporal-mode tomography of PDC
Temporal-mode tomography of PDC

Extended phasematching
\[\downarrow\]
Controllable time-frequency structure

Temporal-mode tomography of PDC

Temporal mode tomography of PDC

All paths to two detectors for $g^{(2)}$ measurement

Purity from the $g^{(2)}$

All paths to two detectors for $g^{(2)}$ measurement

Provides purity information independent of basis

Thermal $g^{(2)} = 2$

Poissonian $g^{(2)} = 1$

Multimode Thermal $g^{(2)} = 1 + \text{Purity}$

A. Christ et al., NJP 13 033027 (2011), arXiv:1012.0262
Temporal-mode tomography of PDC

7-dimensional tomographically complete set

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<td>0.327 ± 0.005</td>
<td>0.317 ± 0.005</td>
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spectral phase matters!
TM tomography of single photons

PDC pump

Parameter estimation with incoherent emitters

* Coherent A.P.E. PulseCheck Autocorrelator
Parameter estimation with incoherent emitters

\[ \text{Rayleigh's Curse} \]

\[ \text{Cramér–Rao Bound, } \text{Var}(\hat{s}) = \frac{4\sigma^2}{N} \int dx \frac{1}{I(x,s)} \left( \frac{\partial I(x,s)}{\partial s} \right)^2 \]

\[ \text{Intensity-Counting Limit} \]
Overcoming the curse

Mode selective measurement break the curse!

\[ \text{Var}(\hat{s}) \geq \frac{1}{N} \int dx \frac{1}{I(x,s)} \left( \frac{\partial I(x,s)}{\partial s} \right)^2 \geq \frac{1}{4N\sigma^2} \]

Intensity-Counting Limit

Quantum Limit

M. Paur, B. Stoklasa, Z. Hradil, L.L. Sanchez-Soto, and J. Rehacek, Optica 3 1144 (2016)
Mode-selective measurement

\[ \frac{P_{HG0}}{P_{HG1}} = \frac{\sigma_{in}^2}{\sigma_{PM}^2} + \frac{s^2}{\sigma_{in}^2} \]

Visibility | Estimator

Changes the time-frequency scale of the curse

Works equally well for time and frequency separations

**Experiment**

Lithium Niobate Type-II, 4.4\(\mu\)m poling

1540 nm + 875 nm → 558 nm
Results: Time-frequency estimation

① Engineered parametric downconversion

② Quantum pulse gate

③ Applications
Thank you for your attention!